Optimizing Snow Plow Routes for the City of Bozeman

Matt Crocker, John Corbett, Brian Locke, Ty Show
Need for Snow Removal

Economic Impact
- Commute times
- Deciding to stay home
- Not contributing to economy
- Increased lead time for deliveries

Safety
- Pedestrians’ safety
- Driver’s safety
- Emergency services
- Property damage
Engineering Design Process

- **Client Need**
  - Qualitative Statements
  - High Level Goals & Constraints
  - Data Collection & Analysis
  - Design Requirements

- **Concept Design**
  - Brainstorming Solutions
  - Solutions already out there
  - Creativity Techniques

- **System-Level Design**
  - Alternative Solution 1
  - Alternative Solution 2
  - Alternative Solution 3
  - Evaluation of Alternatives

- **Detail Design**
  - Best Alternative
  - Detailing of Solution
  - Detailed Analysis tying back to design requirements

- **Documentation**
  - Finalize Deliverables
  - Finishing Detail
Project Scope

The client is interested in optimized routes for:

- Clearing all priority streets in 3-4 hours when a major snow event occurs.
- All proposed solutions should not add to the risk of accidents or increase employee turnover.
Background

City of Bozeman - Precipitation
Population Growth

![Bozeman Population Growth Chart]

- **Year 2011**: 755
- **Year 2012**: 564
- **Year 2013**: 1,143
- **Year 2014**: 1,857
- **Year 2015**: 1,689
- **Year 2016**: 1,866
- **Year 2017**: 1,751
- **Year 2018**: 1,581
Project Objectives

1. Minimize the time required for plowing high priority streets
2. Improve method of route communication to plow operators
Metrics of Success

1. Plow time or projected plow time
2. Cost savings and increases
3. Usability of plow routes
4. Usability of route communication method
Project Constraints

- 8 plows available
- Software should be compatible with city systems
- Limit proposed spending
Current State
Concept Design

- Brainstorming Solutions
- Solutions already out there
- Creativity Techniques
Brainstorm Ideas

Simulation
- Discrete event simulation

Linear Programming
- Mathematically determine optimal routes
Discussions with Faculty

Dr. David Claudio
Associate Professor: Mechanical & Industrial Engineering
Montana State University

Dr. Sean Harris
Instructor: Jake Jabs College of Business & Entrepreneurship
Montana State University
Linear Programming

Optimizing systems

- Helps solve real world problems
- Applied linear algebra
- You can optimize systems that have large amounts of variables

Ex: UPS/Amazon/airlines
Literature Review

Highest priority to peer reviewed journal articles

Routing methods:

- Chinese Postman Problem Variants
- Traveling Salesman Model
- Synchronized Arc Routing
- Genetic Approach to Real Time Vehicle Dispatch
System-Level Design

- Alternative Solution 1
- Alternative Solution 2
- Alternative Solution 3
- Evaluation of Alternatives

Evaluation & Selection
## Design Alternatives

<table>
<thead>
<tr>
<th>Design Alternative</th>
<th>Main Advantage</th>
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<tbody>
<tr>
<td>Windy Postman</td>
<td>Allows for change in path cost</td>
</tr>
<tr>
<td>Traveling Salesman</td>
<td>Relative simplicity</td>
</tr>
<tr>
<td>Synchronized Arc Routing</td>
<td>Allows for multiple plows traveling the same path simultaneously</td>
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Design Requirements

From Established Objectives

1. Can model multiple entities
2. Only requires inputs available from data
3. Compatible software
4. Applicability to the situation
5. Allows for weighting in the objective function
6. Complexity of the model
# Decision Matrix

How well do the design alternatives fit the design requirements?

## Evaluation Matrix for Alternative Modeling Techniques

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<thead>
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### Design Selection

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Simplest Case

\[ t_{ij} = \begin{cases} 
0 & \text{if the plow does not travel from node } i \text{ to node } j \\
1 & \text{if the plow does travel from node } i \text{ to node } j 
\end{cases} \]

\[ c_{ij} = \text{cost to travel from node } i \text{ to node } j \]

Cost matrix

\[ \text{Min } z = \sum_i \sum_j t_{ij} c_{ij} \]

Time

Decision variable

\[ 0 = \sum_i \sum_j t_{ij} \text{ representing arcs in} - t_{ij} \text{ representing arcs out} \quad \forall \text{ nodes} \]

Sets which streets require plowing

\[ 1 = t_{ij} \quad \forall \text{ arcs that require plowing} \]
Adding Turn Penalties

If \( t_{12} = 1 \) and \( t_{24} = 1 \), or \( t_{32} = 1 \) and \( t_{24} = 1 \), then the plow was traveling on road 123 and turned onto road 24.

If \( t_{42} = 1 \) and \( t_{21} = 1 \), or \( t_{42} = 1 \) and \( t_{23} = 1 \), then the plow was traveling on road 24 and turned onto road 123.

\[ \ldots + c_t \times (t_{12} \times t_{24} + t_{32} \times t_{24} + t_{42} \times t_{21} + t_{42} \times t_{23}) \]
Adding Turn Penalties

\[ t_{ijkl} = \begin{cases} 
0 & \text{if plow l does not travel from node } i \text{ to node } j \\
1 & \text{if plow l does travel from node } i \text{ to node } j 
\end{cases} \]

Cost matrix: \( c_{ij} = \text{cost to travel from node } i \text{ to node } j \)

Turn penalty: \( c_t = \text{cost to turn} \)

\[ \min z = \left( \sum_{i} \sum_{j} \sum_{k} \sum_{l} t_{ijkl} * c_{ij} \right) + c_t * (\text{turn penalty terms}) \]

Decision variable

Prevents routes of excessive length

\[ 0 = \sum_{i} \sum_{j} \sum_{k} t_{ijkl} \text{ representing arcs in} - t_{ijkl} \text{ representing arcs out} \quad \forall \text{ nodes} \quad \forall \text{ l} \]

Sets which streets require plowing

\[ \# \text{ of lanes that require plowing} \leq \sum_{k} \sum_{l} t_{ijkl} \quad \forall \text{ arcs} \]

\[ \# \text{ of hours allowable per driver} \geq \sum_{i} \sum_{j} \sum_{k} t_{ijkl} * c_{ij} \quad \forall \text{ l} \]
Final Model Formulation

\[ t_{ijkl} = \begin{cases} 
0 & \text{if plow } l \text{ does not travel from node } i \text{ to node } j \\
1 & \text{if plow } l \text{ does travel from node } i \text{ to node } j 
\end{cases} \]

\[ c_{ij} = \text{cost to travel from node } i \text{ to node } j \]

\[ c_t = \text{cost to turn} \]

\[ \text{Min } z = \left( \sum_i \sum_j \sum_k \sum_l t_{ijkl} \cdot c_{ij} \right) + c_t \cdot (\text{turn penalty terms}) \]

\[ 0 = \sum_i \sum_j \sum_k \sum_l t_{ijkl} - t_{jgkl} \quad \forall \text{ nodes } \forall l \]

\[ 0 \geq \sum_i \sum_j \sum_k t_{ijkl} - \sum_j \sum_k t_{jgkl} \quad \forall \ j \quad \forall \ k \quad \forall \ l \]

\[ \# \text{ of required passes by plows} \leq \sum_k \sum_l t_{ijkl} \quad \forall \text{ arcs} \]

\[ \# \text{ of required passes by graders} \leq \sum_k \sum_l t_{ijk1} + t_{ijk2} \quad \forall \text{ arcs} \]

\[ \# \text{ of hours allowable per driver} \geq \sum_i \sum_j \sum_k t_{ijkl} \cdot c_{ij} \quad \forall \ l \]

\[ 1 = \sum_j t_{270j1l} \quad \forall \ l \]
Running the Model

- For networks of this size, the number of variables will cause problems
- Compromises had to be made to run the model in under 8 hours with 3 Gb of RAM
- The routes obtained as output are not optimal solutions, but are good solutions
Final Optimized Routes

Assuming the longest current route takes 5 hours, the team provided routes that improved on the current routes by approximately 18 minutes or a 6% improvement.
Route Communication Methods

- Heads Up Displays
- GPS units w/ Audio
- Phone Navigation
- Paper Maps
Evaluation of Methods

Route Communication Method Goals:

- Increase the usability of snow plow route communication
- Create countermeasures for line of sight distractions due to current map method
- Decrease distractions due to route communication
- Increase amount of standard operating procedures (SOP’s) for method

Route Communication Method Constraints:

- The countermeasures should be relatively easy to install
- The countermeasures should be low cost
- The new system should stay in line with the overall project goals
- The new system should not make the job harder or less safe

Stakeholder Analysis

Priority Weighting of Design Requirements
Priority Weighting of Design Requirements
Recommendations

GPS with Audio Directions

Cost around $99.99 brand new

Refurbished units are priced in the $60 to $80 range.
Route Communication Implementation Plan

Guide to transition from paper maps to GPS units with Audio directions

Plow Driver Training Session

Test Run of Routes
Conclusion and Takeaways

- The route optimization provided routes that improved on the current routes by approximately 18 minutes or a 6% improvement.
- The use of GPS navigation units with turn-by-turn audio instructions can improve plow operator safety and reduce employee turnover.
Q & A
References

Another One  https://giphy.com/gifs/dj-khaled-GV3aYiEP8qbaoh
Billy  http://memes.ucoz.com/_nw/13/29922471.jpg
Brainstorm  https://www.merriam-webster.com/words-at-play/definition-of-brainstorming
Concept Design  http://www.kaavishinterior.com/ser-concept-designs.html
Right Choice  https://www.shutterstock.com/search/right+choice
Precipitation  https://usclimatedata.com/climate/bozeman/montana/united-states/usmt0040
Bozeman  http://www.montana.edu/marketing/about-msu/images/kg061114-17.jpg
Car Crash  https://i.ytimg.com/vi/vgkOyvabTNE/maxresdefault.jpg